Lab Write-Up Week 6

Abbe Theory of Image Formation

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Conjugate Planes

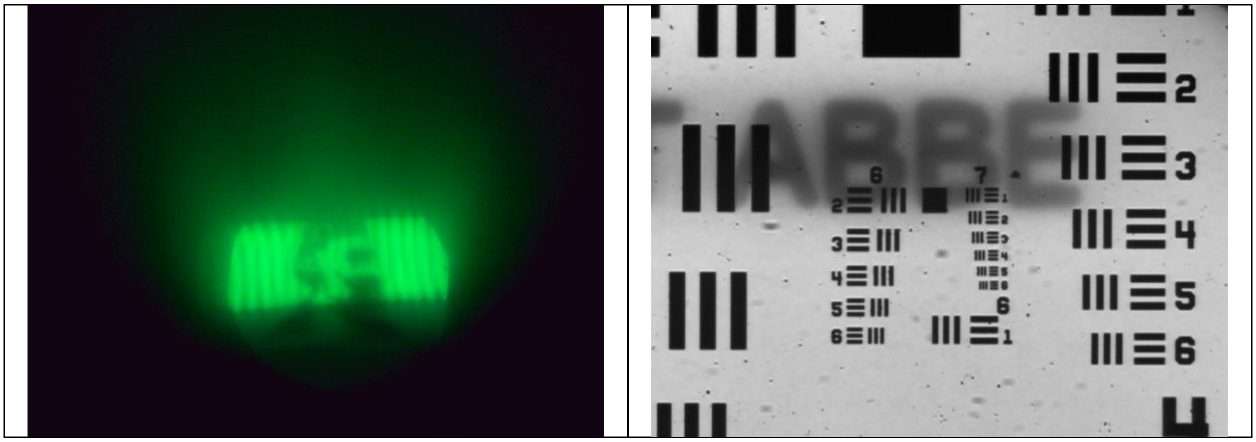

Figure 1. (Left): Image of the Objective BFP. BFP diameter 5.5mm. Exposure 2.001ms. The image of the filament is visible as well as a transparency of Abbe which was placed at aperture stop. (Right): Image of the Sample Plane. BFP diameter 5.5mm. Exposure 118.209ms. The sample as well as the transparency of Abbe's name which was placed at the Field Stop is visible.

The samples of Abbe's head and his name are both in the optical path at the same time, yet do not visibly obstruct each other. This is due to the conjugate planes the samples are placed upon. Abbe's face is placed at the aperture stop which is conjugate to the filament, as well as the Objective BFP and Color camera sensor. As such, the filament, Abbe, and the aperture of the BFP can all be seen in the color image. However, as the system is setup for Kohler Illumination, the filament, and thus all conjugate planes, are imaged at infinity by the condenser lens and thus completely out of focus at the sample plane. As such, the images of Abbe and his name do not interfere with each other.

Abbe Theory

A Ronchi grating sample was imaged using wavelengths of light in the red, blue, and green spectrums. Filters were taken from a Roscolux filter book and the wavelength of light through each filter is as follows.

Color	Peak (nm)	FWHM (nm)
Ir Filter	575	136
Light Red	615	47
Moss Green	540	82
Night Blue	470	68

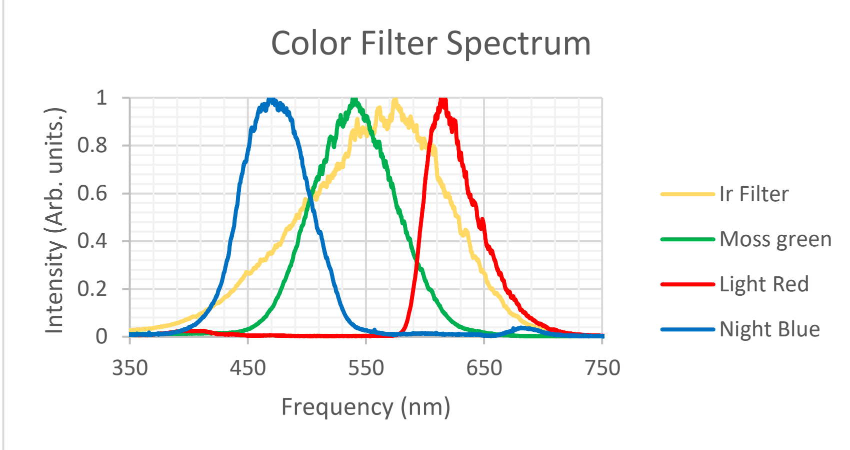

Table 1. Listing of the frequency of light from a Halogen lamp with an IR blocking filter passing through the Red, Green and Blue Roscolux color filters. The color filters have a narrower bandwidth than the unfiltered light.

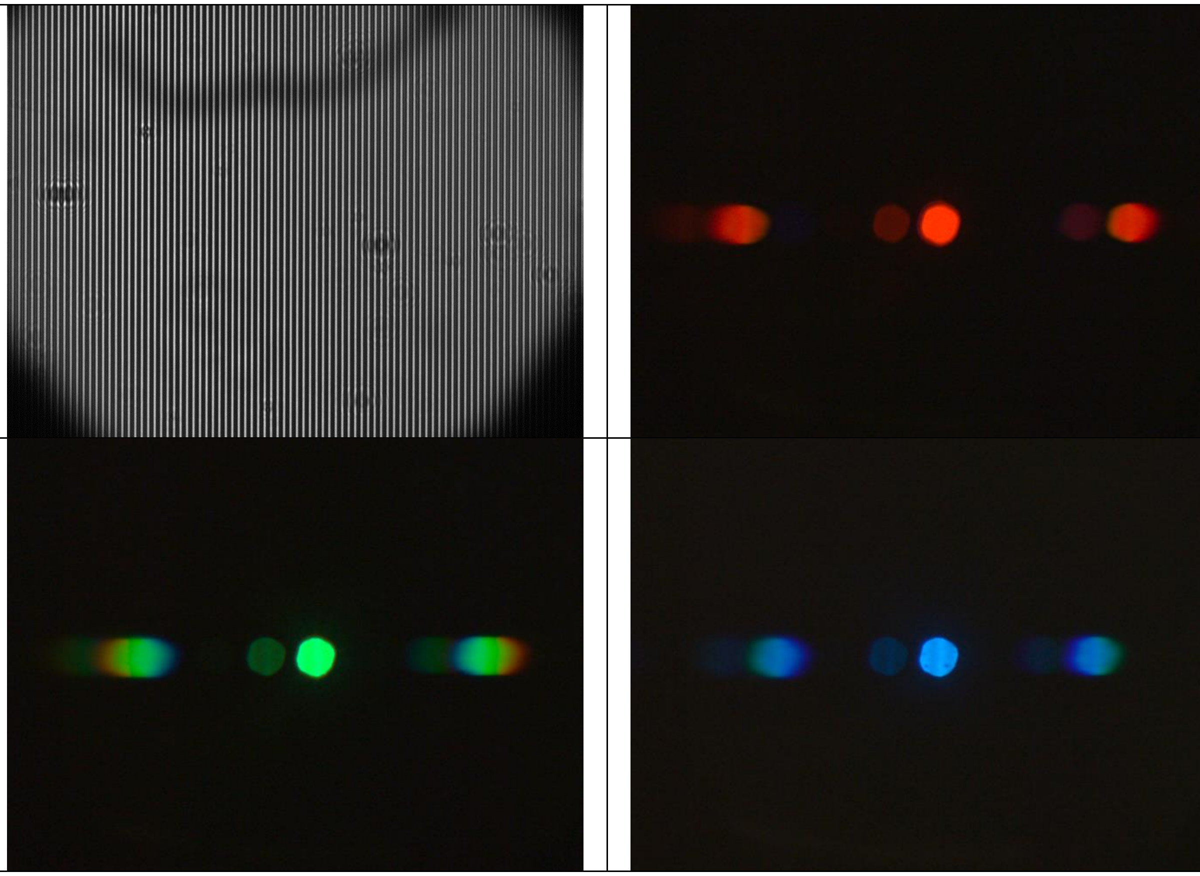

Figure 2. (Top Left): Image of the 200 lp/mm Ronchi grating sample. Exposure 25.512 ms. (Top Right): BFP of the image illuminated with 615nm Red light. (Bottom Left): BFP of the image illuminated with 540nm Green light. (Bottom Right): BFP of the image illuminated with 470nm Blue light.

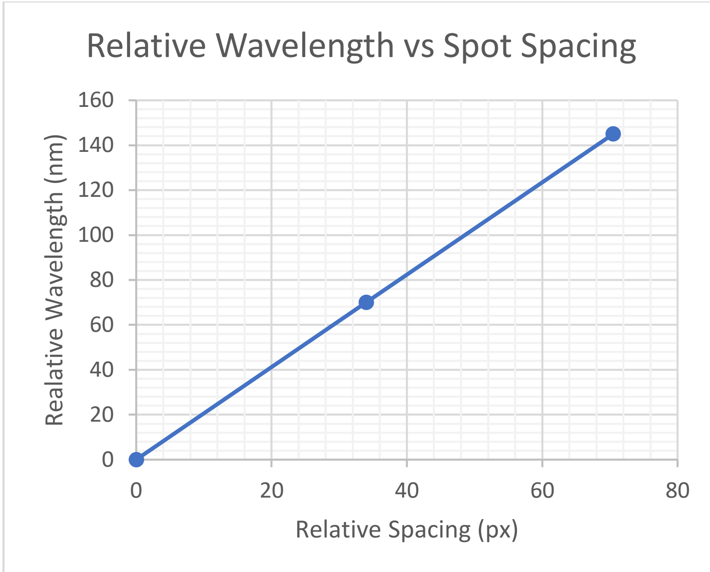

Figure 3. Graph of the Relative Wavelength of light and the pixel spacing between the 0th and 1st maxima. Values are relative to the Wavelength and Interference spacing of blue light. As the wavelength of light increases, so does the spacing between maxima.

The spacing of maxima's through a diffraction grating such as the Ronchi ruling used is defined by

Equation 1: $$\sin\theta = \frac{m\lambda}{d}$$

Where m is the order of the maxima, and d is the distance between slits of the diffraction grating. Θ is the angular position of the maxima. Θ is related to the pixel spacing through a constant amount due to the magnification of the system.

Color	0th to 1st (px)	Wavelength (nm)	relative spacing (px)	relative Wavelength (nm)	Angular Position (rad)	Optical Distance (px)
Light Red	327	615	70.5	145	0.1233	2638
Moss Green	290.5	540	34	70	0.1082	2674
Night Blue	256.5	470	0	0	0.0941	2717

Table 2. Measured spacing between maxima and the calculated angular position and optical distance between sample and objective BFP. The maxima spacing was measuring by taking the distance between the 1st maxima on either side of the 0th maxima and dividing by two to reduce error.

The optical distance of the system can be calculated as the maxima spacing over tan Θ.

As sin Θ is approximately equal to Θ for small angles, and thus linear, the relating between the wavelength of light and the interference spacing is expected to be linear, as seen in Figure 3.

Wavelength and Aperture

To examine the effects of the wavelength of light in the Abbe theory of image formation, a 200lp/mm grating was imaged with green light, with an objective radius just large enough to capture the first maxima. Then The wavelength of light is changed to red and the resultant image viewed.

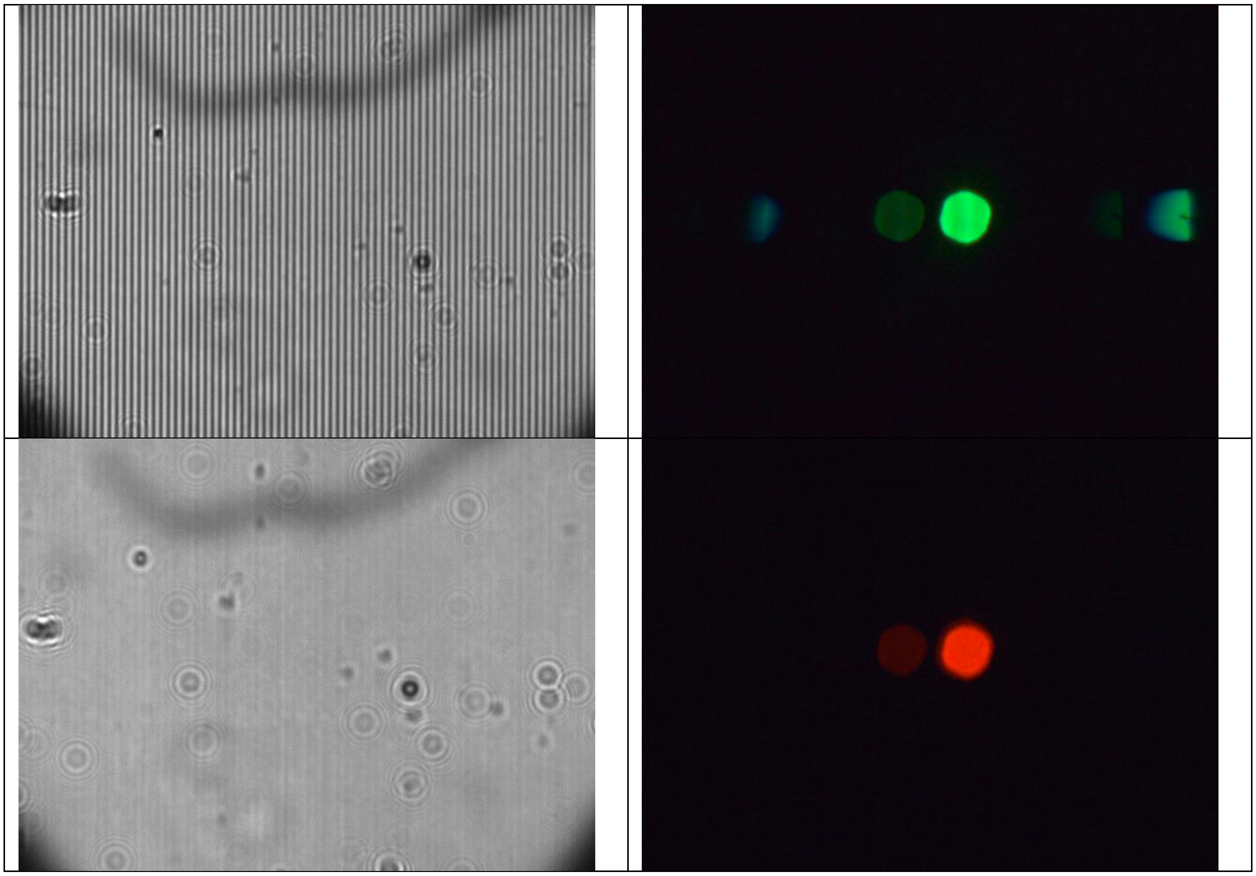

Figure 4. (Top Left): Image of the 200 lp/mm Ronchi grating sample imaged with green 540nm light. Imaged with green 540nm light. Exposure 4ms. (Top Right): BFP image with green light. The first maxima are visible at the edges of the image. (Bottom Left): Image of the 200 lp/mm Ronchi grating sample imaged with red 615nm light. There is no contrast between lines in the grating. (Bottom Right): BFP image with red light. The first maxima are located beyond the radius of the BFP aperture and are not visible in the image.

As evident by the BFP images, a narrow wavelength of light will be diffracted at narrower angles. This allows that, for a given radius objective, more planewaves will enter the lens and thus construct a higher resolution image at the image plane. Thus it is better to image with a lower wavelength of light.

Interdigitated Lines

Two Sets of line pairs (100lp/mm and 200lp/mm) were imaged with monochromatic green light. In the BFP, interference patterns are visible. With the BFP fully open, 5 points are visible, which are the 0^{th} maxima of the 1^{st} maxima of each line set overlayed on top of each other.

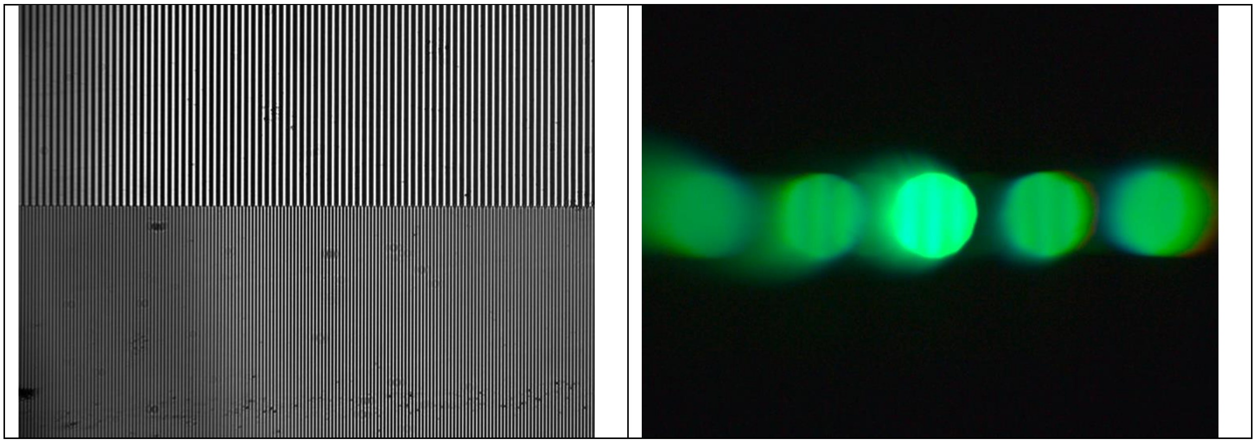

Figure 5. Image plane and BFP of the Interdigitated lines 100lp/mm on top, 200lp/mm on bottom. BFP aperature is open to 7.5mm diameter. Both sets of lines are in contrast. Both sets of maximas are visable in the back focal plane.

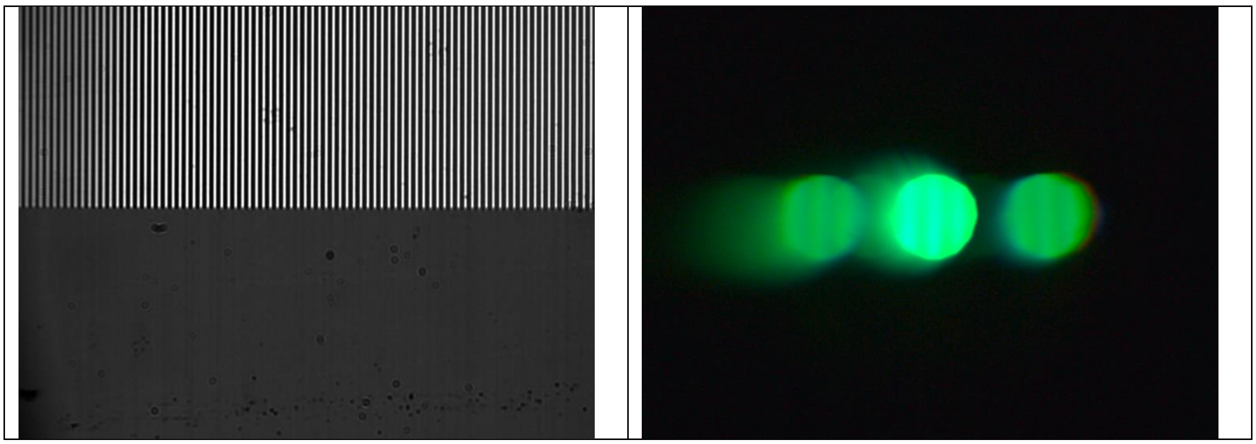

Figure 6. The BFP aperature closed to 4.0mm diameter, such that the 2^{nd} set of maximas are no longer visable in the BFP. Contrast between 200lp/mm lines has been lost, but contrast remains consistant for the 100lp/mm lines.

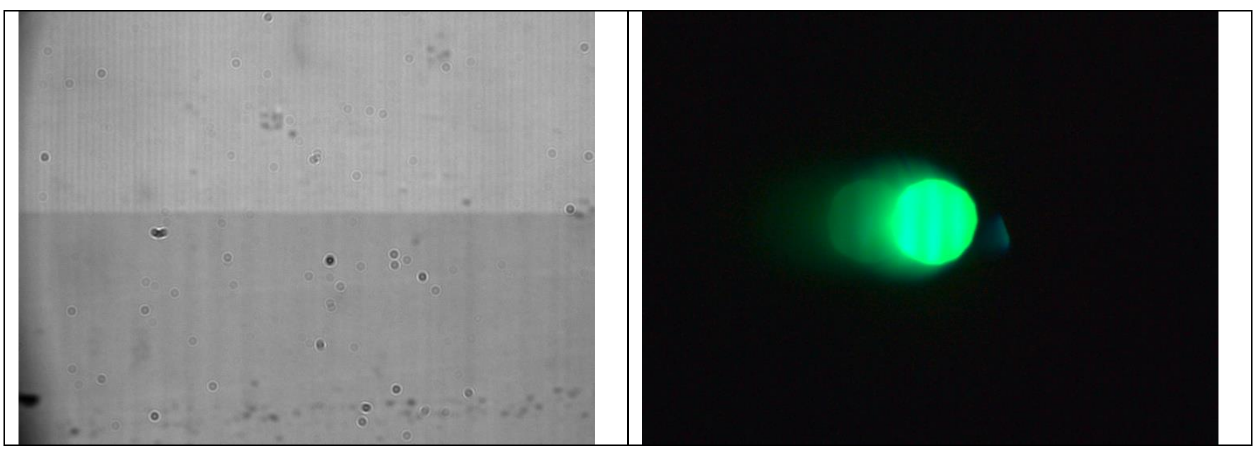

Figure 7. BFP aperature has been closed to 2.0mm diameter such that only the 0^{th} maxima is visable. Both the 100lp/mm and 200lp/mm line gratings are no longer resolveable.

When the outer Maxima are blocked, the 200lp/mm only has 1 maxima so cannot interfere to resolve the image, the 100lp/mm grating still has its 0^{th} and 1^{st} order such that it can be resolved, reducing the BFP aperture to only show the 0^{th} maxima causes both gratings to be unresolvable.

Spatial Filtering

A grid with horizonal and vertical line pairs with a density of 100lp/mm was imaged. Parts of the objective BFP was selectively blocked off to examine the effect on the image.

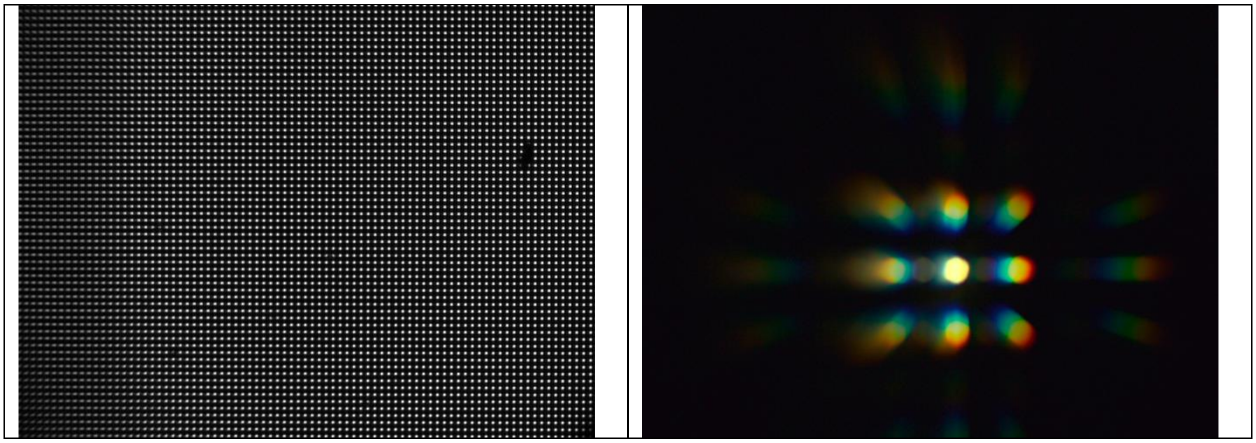

Figure 8. (Left): Image of 100lp/mm grid sample. (Right): BFP image of the interference pattern. The horizontal and vertical lines that compose the sample image create a 2d interreference pattern. The horizontal pattern is apparent as expected with a vertical grating, as well as a vertical pattern as expected with a horizontal grating. There are also patterns that extend into the diagonal direction, caused by the multiplication of the separate transmissions.

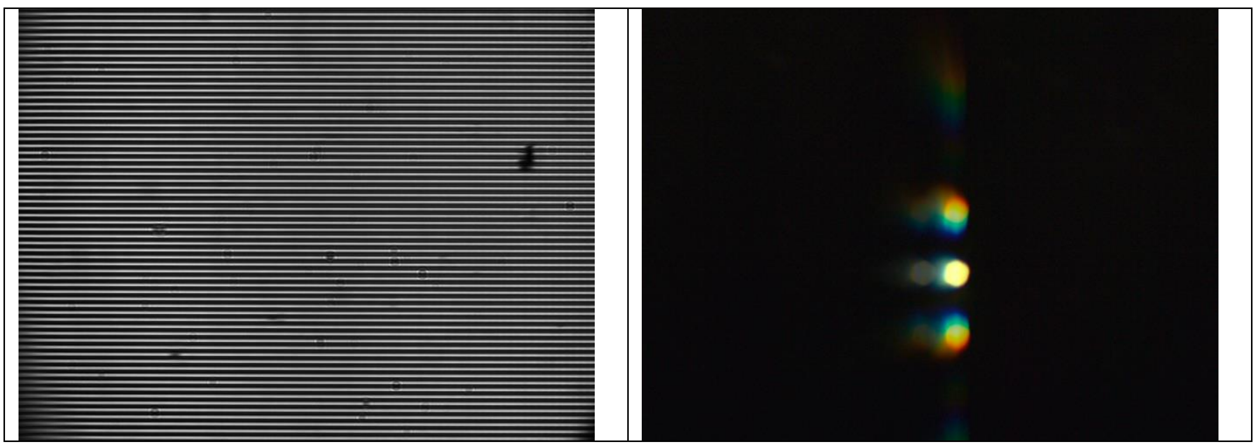

Figure 9. (Left) Resultant image after placing a vertical slit mask at the objective BFP. The Vertical lines of the grid are no longer visible as they were in Figure 8, with only the horizontal lines being visible. (Right): Image of the objective BFP. The mask blocks all but the 0^{th} maxima in the horizontal direction, with the vertical interference patterns passing through unobstructed.

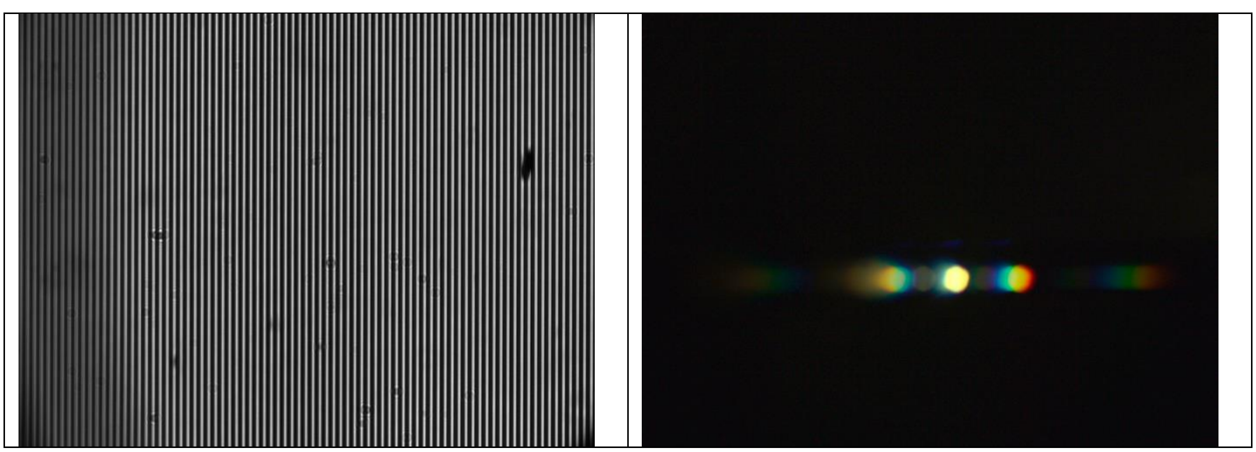

Figure 10. (Left): Resultant image after placing a horizontal slit mask at the objective BFP. The Horizontal lines of the grid are no longer visible as they were in Figure 8, with only the vertical lines being visible. (Right): Image of the objective BFP. The mask blocks all but the 0th maxima in the vertical direction, with the horizontal interference patterns passing through unobstructed.

When light from the illumination sources reaches the sample, the light is diffracted as a series of plane waves at different angles. At the objective BFP the plane waves are focus down to a point of light such that each maximum in the maxima in the interference pattern represent a plane wave. These plane waves interfere at the imaging plane to reconstruct the image. However, when waves are blocked at the objective BFP, the reconstructed image will be missing information and contrast between features will be reduced. With only the 0^{th} maxima, no interference occurs in that direction and thus the contrast between features perpendicular to the axis becomes 0.

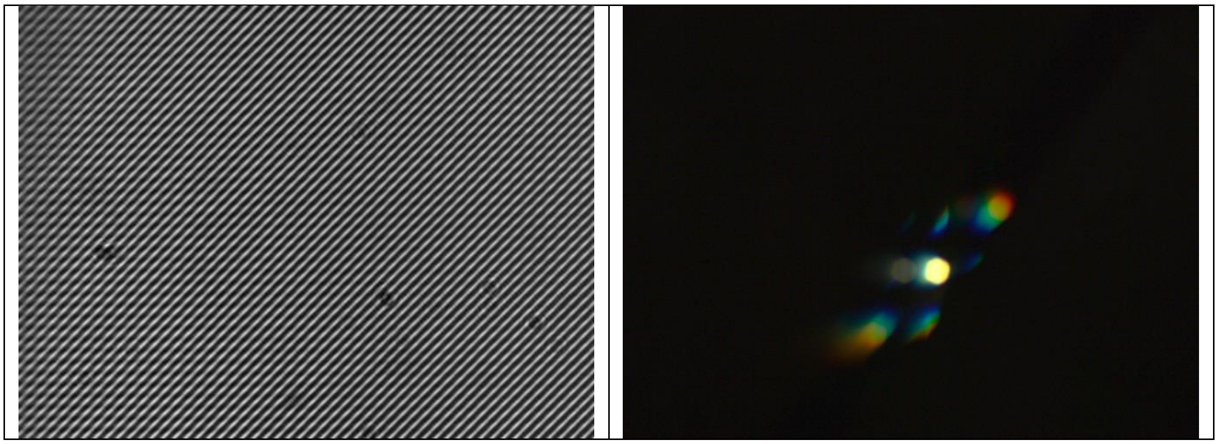

Figure 11. (Left): Resultant image after placing a diagonal slit mask at the objective BFP. Image shows diagonal lines.

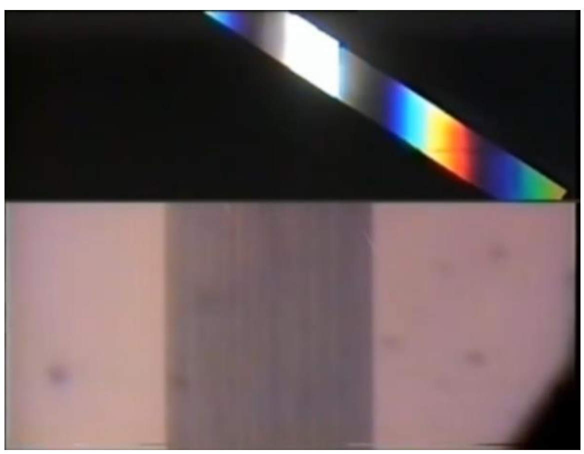

Figure 12. Prof. Evennett's demonstration with a diagonal mask, Sample is a vertical line grating, with the image having no contrast.

The explanation for the difference between the results of Figure 11 and Figure 12. Is that in Figure 11 the condenser aperture stop has been closed to a pinhole size, such that the light through the condenser and thus all light at the objective BFP is spatially coherent. Therefore, all points of light in Figure 11 can interfere to form an image. In Prof. Evennett's demonstration, the condenser must be open further, such that the illumination is composed of multiple illumination sources, incoherent to each other. The diagonal mask blocks the coherent points that are aligned on the horizontal axis, such that the resultant image has no contrast.